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MHD Radiative Heat and Mass Transfer Nanofluid Flow Past a Horizontal Stretching Sheet in a Rotating System

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Abstract- Unsteady radiative MHD heat and mass transfer nanofluid flow through horizontal stretching sheet under the action of strong magnetic field have been investigated. To obtain the non-similar coupled nonlinear momentum, energy and concentration equations, usual non-dimensional variables have been used. The explicit finite difference methods with stability and convergence analysis have been used to solve the obtained numerical solutions of the above problem. The stability and convergence analyses have been used for measuring the restriction of the useful parameters. The obtained numerical results have been presented graphically and discussed in details. Finally, qualitative comparisons of our results with published results have been shown in tabular form.

Index terms: Nanofluid ; MHD; Rotating System; Explicit Finite Difference Method; Stretching Sheet; Unsteady: Radiative

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1. Introduction

Nanofluids have increased thermal conductivity at law nanoparticale concentrations, strong temperature dependent thermal conductivity, and non-linear increase in thermal conductivity with nanoparticale concentration, increase in boiling critical heat flux, these four novel characteristics of nanofluids makes them next generation of flow and heat- transfer fluids. For the heat and mass transfer nanofluids are three-to eight folds better than the conventional fluids. Day by day the applications of nanofluids are increasing. Nanofluids are frequently used in many Engineering works, scientific works and industrial works. Nanofluids are used in microelectronics, fuel cells, hybrid-powered engines, pharmaceuticals process, heat exchangers, vehicle thermal management , nuclear reactor coolant, in grinding, machining, in space technology, defense and ships, ceramic industries, plastic industries , Bio-medical technology etc. The word “nanofluid” was first invented by Choi [1] in order to develop advanced heat transfer fluids with substantially higher conductivities. Wang[2] studies the problem of three dimensional fluid flow due to stretching flat plate.Putra et al.[3] examined that the water based nanofluid containing Al_2O_3 or CuO nanoparticals increased thermal conductivity two-to four folds.Na and Pop[4] studied an unsteady flow past a stretching sheet. Sattar and Alam[5] investigates unsteady free convection and mass transfer flow of a viscous , incompressible and electrically conducting fluid past a moving finite vertical porous plate with thermal diffusion. Buongiorno [6] studies the abnormal increase of thermal conductivity of nanofluids. The effects of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream velocity is investigated Jangid and Tomer[7].Khan and pop[8] have been investigated the problem of laminar boundary layer flow of a nanofluid past a stretching sheet.The boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition in presence of magnetic field and thermal radiation studied by Gbadeyan et al.[9].Ibrahim[10] investigated the radiation effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation.

In the present work, radiative heat and mass transfer flow past a horizontal stretching sheet in a rotating system in presence of strong magnetic field. For solving the non dimensional coupled similar and non similar equations are solved by very well known, reliable and novel explicit finite difference method. Numerical results have presented for the range of Prandtl number, Lewis number, Local Reynold’s number and other well-known parameters which are taken arbitrarily for the fluid.

2. MATHEMATICAL MODEL OF FLOW

Consider an MHD free convection and mass transfer flow of an electrically conducting viscous fluid through a stretching sheet $y=0$ in a rotating system. Considered the Cartesian coordinates x, measured along the stretching surface and y is the coordinate measured normal to the stretching surface and z is the coordinate normal to the stretching surface. The flow is assumed to be in the x direction.

The physical configuration and coordinate system are shown in Fig1.Initially the fluid as well as the stretching sheet is at rest, after the whole system is allowed to rotate with a constant angular velocity R about y-axis. Since the system rotates about y-axis, so we can take R'= (0,-Ω, 0). The temperature and the species concentration at the plate are constantly raised from T_w and C_w to T_∞ and C_∞ respectively, which are there after maintained constant, where T_∞ and C_∞ are the temperature and species concentration of the uniform flow respectively. A uniform magnetic field **B** is imposed to the stretching sheet ($y=0$) to be acting normal to the x-axis which is assumed to be electrically non-conducting.

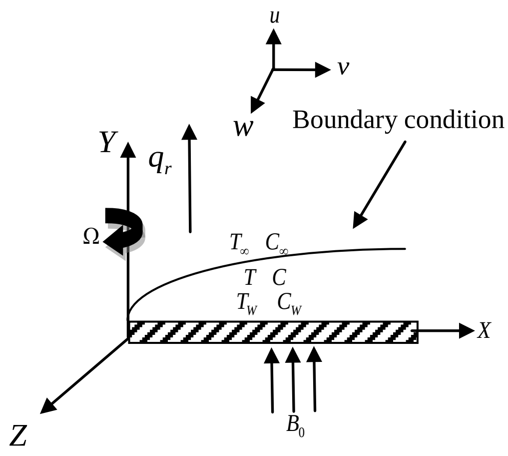

Fig.1.Physical Model of flow

We assumed that **B**= (0, B_0, 0) and the magnetic lines of force are fixed relative to the fluid. q_r is the radiative heat flux acting along the x-axis. Under the usual boundary layer approximation, the MHD unsteady nanofluid flow and heat and mass transfer with the radiation and rotation effect are governed by the following equations.

The continuity equation;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The momentum equation in x-direction;

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+2w\Omega+\frac{\sigma B^2{}_0}{\rho}(U-u) \tag{2}$$

The momentum equation in z-direction;

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\upsilon\left(\frac{\partial^2 w}{\partial y^2}\right)-2u\Omega-\frac{\sigma B^2{}_0 w}{\rho} \tag{3}$$

The energy equation;

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{\alpha}{k}\left(\frac{\partial q_r}{\partial y}\right)+\tau\left[\mathrm{D_B}\left(\frac{\partial \mathrm{C}}{\partial \mathrm{y}}\cdot\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)+\frac{\mathrm{D_T}}{\mathrm{T_\infty}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)^2\right] \tag{4}$$

The concentration equation;

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{\mathrm{D_T}}{\mathrm{T_\infty}}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

The initial and boundary conditions are;

$t=0$, $u_w=U_0=ax,\ \ v=w=0,\ \ T=T_\infty,\ \ \mathrm{C}=\mathrm{C}_\infty,$ evrywhere

$$t \geq 0\ ,\ u = 0, v = 0, w = 0,\quad T = T_\infty,\quad \mathrm{C} = \mathrm{C}_\infty,\ \text{ at } x = 0 \tag{6}$$

$$u = U = bx,\quad v = 0,\quad T = T_w,\quad \mathrm{C} = \mathrm{C}_w,\qquad \text{at y} = 0$$
$$u = 0, v = 0, w = 0,\quad T \to T_\infty,\quad \mathrm{C} \to \mathrm{C}_\infty,\ \text{ as y} \to \infty$$

Where ρ is the density of the fluid , υ is the kinematic viscosity , D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, α is the thermal diffusivity , κ is the thermal conductivity , u_w is the stretching velocity, U is the uniform velocity. The Rosseland approximation is expressed for radiative heat flux and leads to the form as,

$$q_r = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y} \tag{7}$$

Where κ^* is the mean absorption coefficient, σ^* is the Stefan-Boltzmann constant. The temperature difference with in the flow is sufficiently small. So that T^4 may be expressed as a linear function of the temperature, then the Taylor's series for T^4 about T_∞ after neglecting higher order terms,

$$T^4 \cong 4T\,T_\infty^4 - 3T_\infty^3 \tag{8}$$

The dimensionless variables that are using in the equations (1)-(5) are as follows;

$$X = \frac{xU_0}{\upsilon},\quad Y = \frac{yU_0}{\upsilon},\quad U = \frac{u}{U_0},\quad V = \frac{v}{U_0},\quad W = \frac{w}{U_0},\quad \tau = \frac{tU_0^2}{\upsilon} \tag{9}$$

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{10}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{1}{R_e}\left(\frac{b^2}{a^2}\right) + \frac{\partial^2 U}{\partial Y^2} + 2R'W + M(1-U) \tag{11}$$

$$\frac{\partial W}{\partial \tau} + U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} = \frac{\partial^2 W}{\partial Y^2} - 2R'U - MW \tag{12}$$

$$\frac{\partial \overline{T}}{\partial \tau} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \left(\frac{1+R}{P_r}\right)\frac{\partial^2 \overline{T}}{\partial Y^2} + N_b\left(\frac{\partial \overline{T}}{\partial Y}\cdot\frac{\partial \overline{C}}{\partial Y}\right) + N_t\left(\frac{\partial \overline{T}}{\partial Y}\right)^2 \tag{13}$$

$$\frac{\partial \overline{C}}{\partial \tau} + U\frac{\partial \overline{C}}{\partial X} + V\frac{\partial \overline{C}}{\partial Y} = \frac{1}{L_e}\left[\frac{\partial^2 \overline{C}}{\partial Y^2} + \left(\frac{N_t}{N_b}\right)\frac{\partial^2 \overline{T}}{\partial Y^2}\right] \tag{14}$$

The non-dimensional boundary condition's are;

$$\tau \leq 0,\quad U = 0,\quad V = 0,\quad W = 0,\quad \overline{T} = 0,\quad \overline{C} = 0, \text{ Everywhere}$$
$$\tau > 0,\quad U = 0,\quad V = 0,\quad W = 0,\quad \overline{T} = 0,\quad \overline{C} = 0, \text{ At } X=0 \tag{15}$$
$$U = 1,\quad V = 0,\quad W = 0,\quad \overline{T} = 1,\quad \overline{C} = 1, \text{ At } Y=0$$
$$U = 0,\quad V = 0,\quad W = 0,\quad \overline{T} \to 0,\quad \overline{C} \to 0,\ \text{ as } Y \to \infty \tag{16}$$

The non-dimensional quantities are;

$M = \dfrac{\sigma B_0^2 \upsilon}{\rho U_0^2}$ (Magnetic parameter), $R = \dfrac{16\sigma^* T_\infty^3}{3kk^*}$ (Radiation parameter), $P_r = \dfrac{\upsilon}{\alpha}$ (Prandtl number)

$N_b = \dfrac{\tau D_B (C_w - C_\infty)}{\upsilon}$ (Brownian parameter), $N_t = \dfrac{D_T}{T_\infty}\dfrac{\tau}{\upsilon}(T_w - T_\infty)$ (Thermophoresis parameter),

$R_e = \dfrac{xu_w}{\upsilon}$ (Local Reynold's number), $R' = \dfrac{\Omega\upsilon}{U_0^2}$, $L_e = \dfrac{\upsilon}{D_B}$ (Lewis number)

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(Rotational Velocity), and $\frac{b}{a}$ (Stretching Parameter).

3. NUMERICAL SOLUTIONS

In order to solve the non-similar coupled nonlinear, non-dimensional partial differential equations, by the explicit finite difference method, it is required a set of finite difference equations. For this, a rectangular flow field is chosen and the region is divided into a grid of lines parallel *X, Y* and *Z* axes, where *X*- axis is taken along the stretching sheet, *Y* and *Z*- axis are normal to the stretching sheet.

Here we consider the height of the stretching sheet $X_{\max}$ (=100) i. e. *X* varies from 0 to 100 and assumed $Y_{\max}$(=25) as corresponding to $Y \to \infty$ i.e.*Y* varies from 0 to 25.There are *m*(=125) and *n*(=125) grid spacing in the *X* and *Y* directions respectively as shown in Fig 2.It is assumed that ΔX, ΔY are constant mesh sizes along *X* and *Y* directions respectively and taken as follows, $\Delta X = 0.8(0 \le X \le 100)$, and $\Delta Y = 0.2(0 \le Y \le 25)$ with the smaller time-step, $\Delta\tau = 0.005$. Let U', V', W', $\overline{T}'$ and $\overline{C}'$ denote the values of U, V, W, $\overline{T}$ and $\overline{C}$ at the end of the time step respectively. Using the finite difference approximation, we obtain the following appropriate set of finite difference equations; Let U', V', W', $\overline{T}'$ and $\overline{C}'$ denote the values of U, V, W, $\overline{T}$ and $\overline{C}$ at the end of the time step respectively. Using the finite difference approximation, we obtain the following appropriate set of finite difference equations;

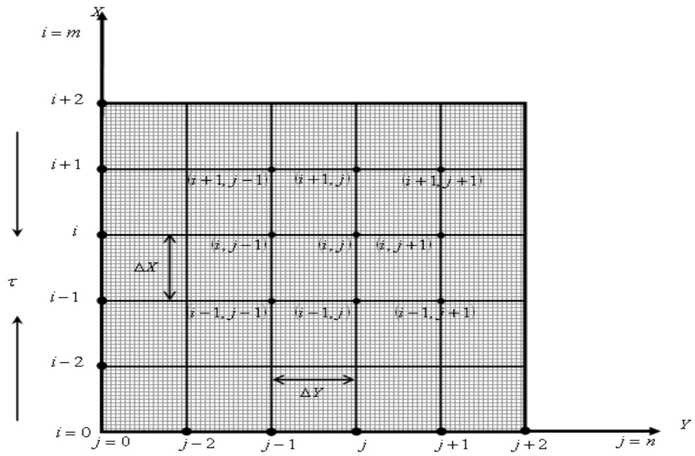

Fig. 2. Explicit finite difference system grid.

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta X} + \frac{V'_{i,j} - V'_{i,j-1}}{\Delta Y} = 0 \tag{17}$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \frac{1}{R_e}\left(\frac{b^2}{a^2}\right) + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + 2R'W_{i,j} + M\left(1 - U_{i,j}\right) \tag{18}$$

$$\frac{W'_{i,j} - W_{i,j}}{\Delta\tau} + U_{i,j}\frac{W_{i,j} - W_{i-1,j}}{\Delta X} + V_{i,j}\frac{W_{i,j+1} - W_{i,j}}{\Delta Y} = \frac{W_{i,j+1} - 2W_{i,j} + W_{i,j-1}}{(\Delta Y)^2} - 2R'U_{i,j} - MW_{i,j} \tag{19}$$

$$\frac{\overline{T}'_{i,j} - \overline{T}_{i,j}}{\Delta\tau} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \left(\frac{1+R}{P_r}\right)\left(\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2}\right) + N_b\left(\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y}\cdot\frac{\overline{C}_{i,j+1} - \overline{C}_{i,j}}{\Delta Y}\right) + N_t\left(\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y}\right)^2 \tag{20}$$

$$\frac{\overline{C}'_{i,j}-\overline{C}_{i,j}}{\Delta\tau}+U_{i,j}\frac{\overline{C}_{i,j}-\overline{C}_{i-1,j}}{\Delta X}+V_{i,j}\frac{\overline{C}_{i,j+1}-\overline{C}_{i,j}}{\Delta Y}=\frac{1}{L_e}\left[\left(\frac{\overline{C}_{i,j+1}-2\overline{C}_{i,j}+\overline{C}_{i,j-1}}{(\Delta Y)^2}\right)+\left(\frac{N_t}{N_b}\right)\left(\frac{\overline{T}_{i,j+1}-2\overline{T}_{i,j}+\overline{T}_{i.j-1}}{(\Delta Y)^2}\right)\right] \quad (21)$$

With initial and boundary conditions

$$U_{i,j}^{0}=0,\ V_{i,j}^{0}=0,\ W_{i,j}^{0}=0,\ \overline{T}_{i,j}^{0}=0,\ \overline{C}_{i,j}^{0}=0 \quad (22)$$

$$U_{0,j}^{n}=0,\ V_{0,j}^{n}=0,\ W_{0,j}^{n}=0,\ \overline{T}_{0,j}^{n}=0,\ \overline{C}_{0,j}^{n}=0$$

$$U_{i,0}^{n}=1,\ V_{i,0}^{n}=0,\ W_{i,0}^{n}=0,\ \overline{T}_{i,0}^{n}=1,\ \overline{C}_{i,0}^{n}=1$$

$$U_{i,L}^{n}=0,\ V_{i,L}^{n}=0,\ W_{i,L}^{n}=0,\ \overline{T}_{i,L}^{n}=0,\ \overline{C}_{i,L}^{n}=0,\quad \text{where } L\to\infty \quad (23)$$

Here the subscripts *i* and *j* designate the grid points with *X* and *Y* coordinates respectively and the subscript n represents a value of time, $\tau = n\Delta\tau$ where *n*=0, 1, 2, 3……The stability conditions of the method are

$$U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}-\frac{\Delta\tau}{2R_eU}\left(\frac{b^2}{a^2}\right)-\frac{\Delta\tau}{2U}(1-U)\le 1 \quad (24)$$

$$U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}+\frac{M\,\Delta\tau}{2}\le 1 \quad (25)$$

$$U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+2\frac{\Delta\tau}{(\Delta Y)^2}\left(\frac{1+R}{P_r}\right)+2N_b\frac{\Delta\tau}{(\Delta Y)^2}\overline{C}+2N_t\frac{\Delta\tau}{(\Delta Y)^2}\overline{T}\le 1 \quad (26)$$

$$U\frac{\Delta\tau}{\Delta X}+|V|\frac{\Delta\tau}{\Delta Y}+\frac{2}{L_e}\frac{\Delta\tau}{(\Delta Y)^2}\le 1 \quad (27)$$

Since from the initial condition, $U=V=\overline{T}=\overline{C}=0$ at $\tau=0$ and the consideration due to stability and convergence analysis is $E_c<1$ and $R\ge 0.10$ Hence convergence criteria of the method are $P_r \ge 0.73$, and $N_b \ge 0.10, N_t \ge 0.10.$

4. RESULTS AND DISCUSSION

In order to investigate the physical representation of the problem, the numerical values of temperature and species concentration within the boundary layer have been computed for different values of Magnetic parameter *M*, Radiation parameter *R*, Prandtl number P_r ,Reynolds number R_e ,Lewis number L_e ,Brownian motion N_b ,and Thermophoresis parameter N_t ,Rotational velocity R' ,To obtain the steady-state solutions of the computation, the calculation have been carried out up to non-dimensional time $\tau = 5$ to 80. It is observed that the numerical values of $U, W, \overline{T}$ and $\overline{C}$ however, show little changes after $\tau = 50$. Hence at $\tau = 50$ the solutions of all variables are steady-state solutions. In Fig.3 it represents that the temperature distribution increases with the increases with the increase of radiation parameter and the temperature distribution decreases with the increase of Prandtl number. In Fig.4 the temperature distribution increases with the increase of Eckert number and the temperature distribution increases with the increase of Brownian motion parameter. The graphical presentation in fig.5 (a) shows that the temperature distribution increases with the increase of Thermophoresis parameter and in fig.5 (b) shows that the concentration distribution decreases with the increase of Lewis number.Fig.6(a) represents that at first the concentration distribution increases with the increase of Radiation parameter but after some time the concentration distribution decreases with the increase of Radiation parameter again reverse effect is shown after some time. Fig.6 (b) represents that at first the concentration distribution decreases with the increase of Prandtl number but after some time the concentration distribution increases with the increase of Prandtl number.Fig.7 (a) indicates that the concentration distribution decreases with the increase of Brownian motion parameter. Fig.7 (b) represents that at first the concentration distribution decreases with the increase of Thermophoresis parameter but after some time it is shown reverse effect.

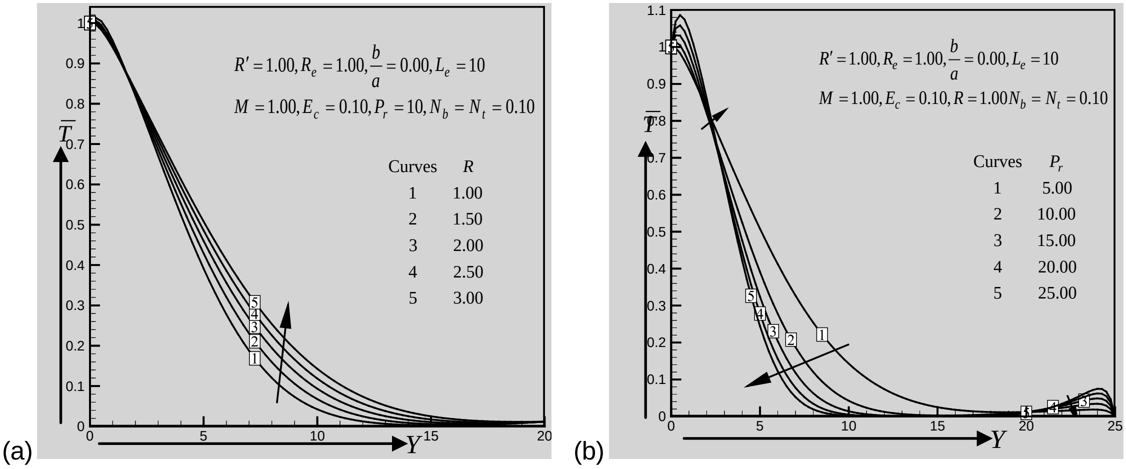

Fig.3.(a)The temperature profile for the different values of Radiation parameter and (b)Prandtl number.

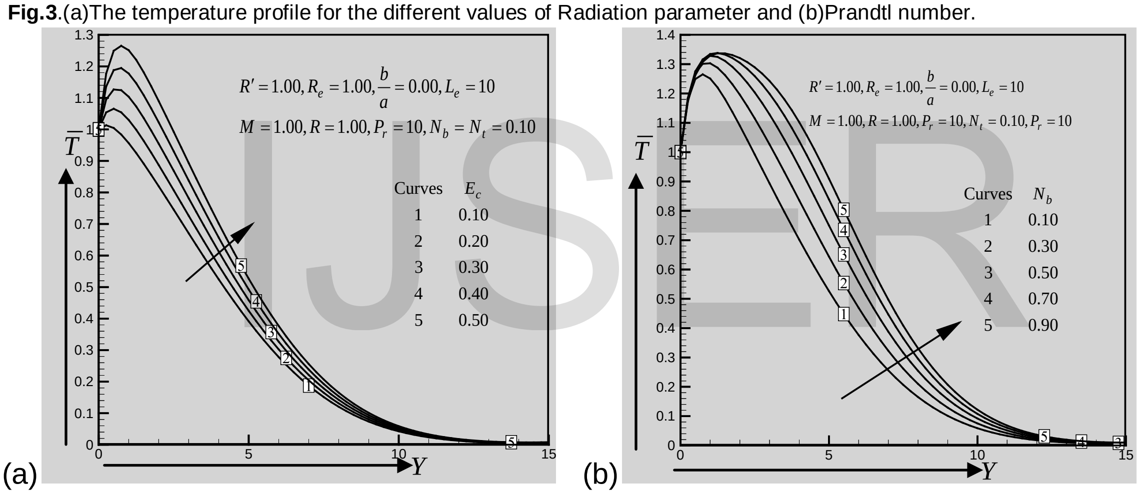

Fig.4.(a) The temperature profile for different values of Eckert number and(b) Brownian motion parameter.

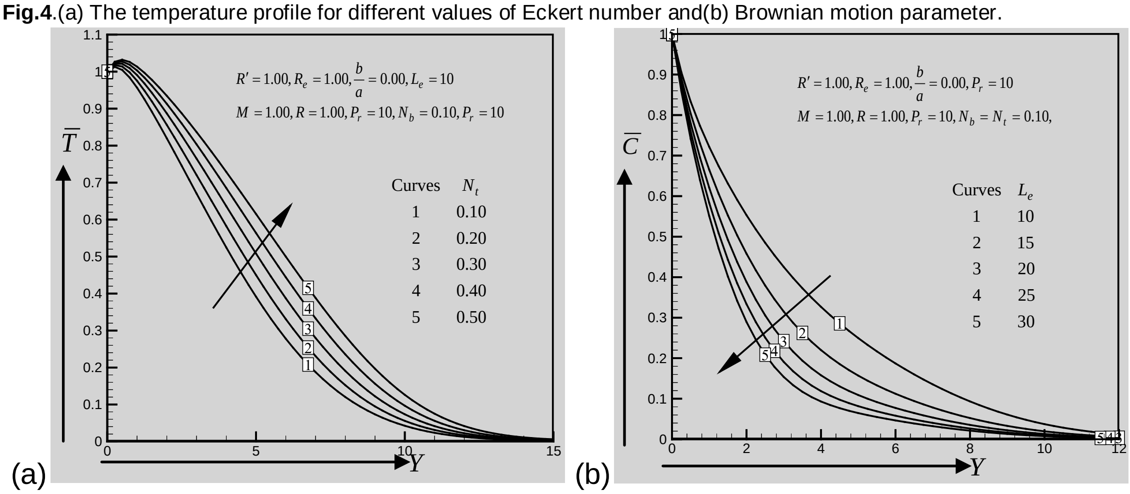

Fig.5. (a) Temperature profile for different values of Thermophoresis parameter and (b) the concentration profile for the different values of Lewis number.

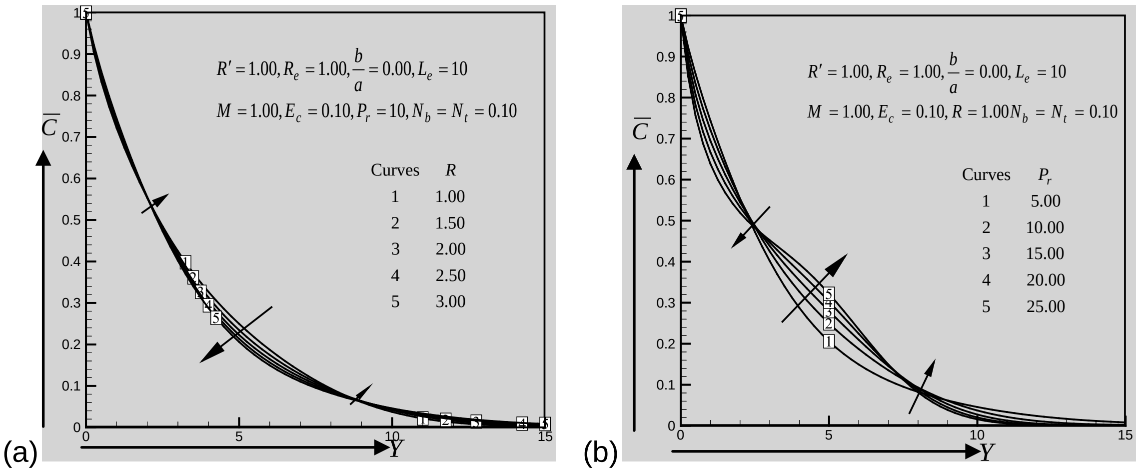

Fig.6. (a) The concentration profile for different values of Radiation parameter and (b) Prandtl number.

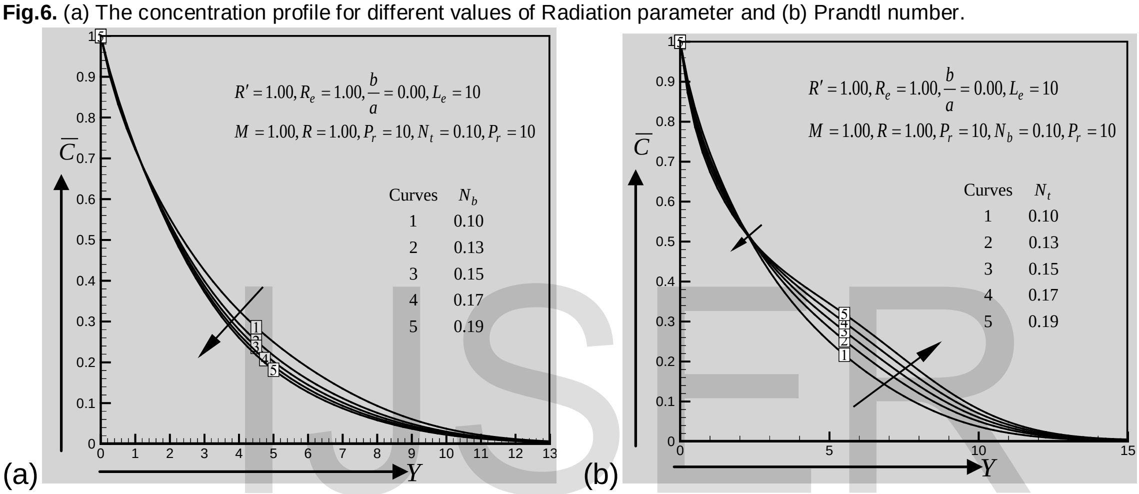

Fig.7. (a) The concentration profile for different values of Brownian motion parameter and (b)Thermophoresis parameter.

5. CONCLUSIONS

1. For increasing the Brownian and Thermophoresis parameter, temperature distribution increases where as the concentration distribution decreases for increasing the Brownian parameter.
2. Thermal boundary layer thickness decreases for increasing Prandtl number and concentration boundary layer thickness decreases for increasing Lewis number.
3. The MHD and Radiation effect through the boundary layer for both temperature and concentration has a great impact on flow pattern. As the Radiation parameter increases then the temperature distributions gradually increases while the reverse effects seen for concentration distributions.

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